Integrating Planning and Real-Time Operations in Public Transport Systems

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Outline

Outline

Timetabling based on the set of all planned trips:

1

$$q_t \in \{0,1\} \quad t \in V_r^{TT},\ r \in R,$$

Timetabling based on the set of all potential trips:

2

$$\sum_{(j,i)\in A_r^{TT}} z_{ji} - \sum_{(i,j)\in A_r^{TT}} z_{ij} = \begin{cases} -1, & \text{if } i = v_r^-, \\ 1, & \text{if } i = v_r^+, \\ 0, & \text{else,} \end{cases} \qquad i \in V_r^{TT},\ r \in R.$$

$$z_{ij} \in \{0,1\} \qquad (i,j) \in A_r^{TT},\ r \in R,$$

Linking constraints TTVSDS:

1

$$\sum_{d\in D}\sum_{e\in E_d^{pair}}\sum_{(i,j)\in A_e^{VSDS}(t)} x_{ij}^e = 1 - q_t \qquad t \in \hat{T}_r^{RTW},\ r \in R,$$

2

$$\sum_{d\in D}\sum_{e\in E_d^{pair}}\sum_{(i,j)\in A_e^{VSDS}(t)} x_{ij}^e = \sum_{(t,k)\in A_r^{TT}} z_{tk} \qquad t \in T_r,\ r \in R,$$

3

$$\sum_{d\in D}\sum_{e\in E_d^{pair}}\sum_{u\in U_e(t)} x_u^e = 1 - q_t \qquad t \in \hat{T}_r^{RTW},\ r \in R,$$

4

$$\sum_{d\in D}\sum_{e\in E_d^{pair}}\sum_{u\in U_e(t)} x_u^e = \sum_{(t,k)\in A_r^{TT}} z_{tk} \qquad t \in V_r^{TT},\ r \in R,$$

Vehicle-Driver pair Scheduling Compact (with optional time-window arc constraints):

1

$$\sum_{(j,i)\in A_e^{VSDS}} x_{ji}^e - \sum_{(i,j)\in A_e^{VSDS}} x_{ij}^e = \begin{cases} -1, & \text{if } i = s^-, \\ 1, & \text{if } i = s^+, \\ 0, & \text{else,} \end{cases}$$

$$i \in V_e^{VSDS},\ e \in E_d^{pair},\ d \in D,$$

$$\sum_{d\in D}\sum_{e\in E_d^{pair}}\sum_{(i,j)\in A_d^{VSDS}(q)} x_{ij}^e \le w_q \qquad q \in Q,$$

$$W_1^{se} \le \sum_{(i,j)\in A_e^{VSDS}} w_{ij}^s x_{ij}^e \le W_2^{se} \qquad s \in S_e,\ e \in E_d^{pair},\ d \in D,$$

$$x_{ij}^e \in \{0,1\} \qquad (i,j) \in A_e^{VSDS},\ e \in E_d^{pair},\ d \in D.$$

a

$$x_{ki}^e \le x_{ij}^e \qquad (k,i) \in A_e^{TW}, (i,j) \in A_e^{VSDS}(t), t \in \hat{T}_e^{RTW}, e \in E_d^{pair}, d \in D,$$

$$x_{ki}^e \le x_{jl}^e \quad (k,i) \in A_e^{TW}, (i,j) \in A_e^{VSDS}(t), (j,l) \in A_e^{PB}, t \in \hat{T}_e^{RTW}, e \in E_d^{pair}, d \in D,$$

Vehicle-Driver pair Scheduling Set Partitioning:

2

$$\sum_{u\in U_d(e)} x_u^d = 1, \qquad e \in E_d^{pair},\ d \in D,$$

$$\sum_{d\in D}\sum_{u\in U_d(q)} x_u^d \le w_q \qquad q \in Q.$$

$$x_u^d \in \{0,1\} \qquad u \in U_d,\ d \in D,$$

Timetabling based on the set of all planned trips:

[1]

$$q_t \in \{0,1\} \quad t \in V_r^{TT},\ r \in R,$$

Timetabling based on the set of all potential trips:

[2]

$$\sum_{(j,i)\in A_r^{TT}} z_{ji} - \sum_{(i,j)\in A_r^{TT}} z_{ij} = \begin{cases} -1, & \text{if } i = v_r^-, \\ 1, & \text{if } i = v_r^+, \\ 0, & \text{else}, \end{cases} \qquad i \in V_r^{TT},\ r \in R.$$

$$z_{ij} \in \{0,1\} \qquad (i,j) \in A_r^{TT},\ r \in R,$$

Linking constraints TTVS:

[1]

$$\sum_{d\in D(t)} \sum_{(i,j)\in A_d^{VS}(t)} x_{ij}^d = 1 - q_t \qquad t \in \hat{T}_r^{RTW},\ r \in R,$$

[2]

$$\sum_{d\in D} \sum_{e\in E_d^{pair}(t)} x_{ij}^e = \sum_{(t,k)\in A_r^{TT}} z_{tk} \qquad t \in T_r,\ r \in R,$$

[3]

$$\sum_{d\in D} \sum_{u\in U_d(t)} x_u^d = 1 - q_t \qquad t \in \hat{T}_r^{RTW},\ r \in R,$$

[4]

$$\sum_{d\in D} \sum_{u\in U_d(t)} x_u^d = \sum_{(t,k)\in A_r^{TT}} z_{tk} \qquad t \in V_r^{TT},\ r \in R,$$

Vehicle Scheduling Compact (with optional time-window arc constraints):

[1]

$$\sum_{(j,i)\in A_d^{VS}} x_{ji}^d - \sum_{(i,j)\in A_d^{VS}} x_{ij}^d = \begin{cases} -S^-, & \text{if } i = s^-, \\ S^+, & \text{if } i = s^+, \\ 0, & \text{else}, \end{cases} \qquad i \in V_d^{VS},\ d \in D,$$

$$\sum_{(j,i)\in A_f^{VS}} x_{ji}^f - \sum_{(i,j)\in A_f^{VS}} x_{ij}^f = \begin{cases} -1, & \text{if } i = s^-, \\ 1, & \text{if } i = s^+, \\ 0, & \text{else}, \end{cases} \qquad i \in V_f^{VS},\ f \in F_d^{ref},\ d \in D,$$

$$\sum_{d\in D} \sum_{(i,j)\in A_d^{VS}(q)} x_{ij}^d + \sum_{d\in D} \sum_{f\in F_d^{ref}} \sum_{(i,j)\in A_f^{VS}(q)} x_{ij}^f \le w_q \qquad q \in Q.$$

$$x_{ij}^d \le w_{ij}^d,\ x_{ij}^d \in \mathbb{N}_{\ge 0} \qquad (i,j) \in A_d^{VS},\ d \in D.$$

$$x_{ij}^f \in \{0,1\} \qquad (i,j) \in A_f^{VS},\ f \in F_d^{ref},\ d \in D.$$

[a]

$$x_{ki}^d \le x_{ij}^d \qquad (k,i) \in A_d^{TW},\ (i,j) \in A_d^{VS}(t),\ t \in \hat{T}_d^{RTW},\ d \in D,$$

$$x_{ki}^d \le x_{ij}^d \quad (k,i) \in A_d^{TW},\ (i,j) \in A_d^{VS}(t),\ (j,l) \in A_d^{PB},\ t \in \hat{T}_d^{RTW},\ d \in D,$$

Vehicle Scheduling Set Partitioning:

[2]

$$\sum_{u\in U_d(s^-)} x_u^d = S^-, \qquad (s^-, S^-) \in S_d^-,\ d \in D,$$

$$\sum_{u\in U_d(s^+)} x_u^d = S^+, \qquad (s^+, S^+) \in S_d^+,\ d \in D,$$

$$\sum_{u\in U_d(f)} x_u^d = 1, \qquad f \in F_d^{ref},\ d \in D,$$

$$\sum_{d\in D} \sum_{u\in U_d(q)} x_u^d \le w_q \qquad q \in Q.$$

$$x_u^d \in \{0,1\} \qquad u \in U_d,\ d \in D,$$

Linking constraints VSDS:

[1]

$$\sum_{e\in E_d(i,j)} y_{ij}^e = x_{ij}^d \qquad (i,j) \in A_d^{VSDS},\ d \in D,$$

[2]

$$\sum_{e\in E_d(i,j)} y_{ij}^e = \sum_{u\in U_d(i,j)} x_u^d \qquad (i,j) \in A_d^{VSDS},\ d \in D,$$

[3]

$$\sum_{e\in E_d} \sum_{u\in U_e(i,j)} y_u^e = x_{ij}^d \qquad (i,j) \in A_d^{VSDS},\ d \in D,$$

[4]

$$\sum_{e\in E_d} \sum_{u\in U_e(i,j)} y_u^e = \sum_{u\in U_d(i,j)} x_u^d \qquad (i,j) \in A_d^{VSDS},\ d \in D,$$

Driver Scheduling Compact with optional time-window arc constraints:

[1]

$$\sum_{(j,i)\in A_e^{DS}} y_{ji}^e - \sum_{(i,j)\in A_e^{DS}} y_{ij}^e = 0 \qquad i \in V_e^{DS},\ e \in E_d,\ d \in D,$$

$$W_1^{se} \le \sum_{(i,j)\in A_e^{DS}} w_{ij}^s y_{ij}^e \le W_2^{se} \qquad s \in S_e,\ e \in E_d,\ d \in D,$$

$$y_{ij}^e \in \{0,1\} \qquad (i,j) \in A_e^{DS},\ e \in E_d,\ d \in D,$$

[a]

$$y_{ki}^e \le y_{ij}^e \qquad (k,i) \in A_e^{TW},\ (i,j) \in A_e^{DS}(t),\ t \in \hat{T}_e^{RTW},\ e \in E_d,\ d \in D,$$

$$y_{ki}^e \le y_{ij}^e \quad (k,i) \in A_e^{TW},\ (i,j) \in A_e^{DS}(t),\ (j,l) \in A_e^{PB},\ t \in \hat{T}_e^{RTW},\ e \in E_d,\ d \in D,$$

Driver Scheduling Set Partitioning:

[2]

$$\sum_{u\in U_d(e)} y_u^d = 1, \qquad e \in E_d,\ d \in D,$$

$$y_u^e \in \{0,1\} \qquad u \in U_e,\ e \in E_d,\ d \in D.$$

Linking constraints TSDS (graph considering constrained vehicle changes):

[1]

$$\sum_{d\in D} \sum_{e\in E_d} \sum_{(i,j)\in A_e^{DS}} y_{ij}^e = \sum_{(t,k)\in A_r^{TT}} z_{tk} \qquad i \in V^{DS}(t),\ t \in V_r^{TT},\ r \in R,$$

[2]

$$\sum_{d\in D} \sum_{e\in E_d} \sum_{(i,j)\in A_e^{DS}} \sum_{u\in U_e(i,j)} y_u^e = \sum_{(t,k)\in A_r^{TT}} z_{tk} \qquad i \in V^{DS}(t),\ t \in V_r^{TT},\ r \in R,$$

Outline

ALNS: Destroy Operators

Random: A random set of vehicle and driver pairs is selected.

Less frequent choices: The least frequently chosen pairs of the previous iterations are selected.

Line direction: Select pairs that contain trips of a randomly chosen direction are selected. This is done to enable shifting of the corresponding trips.

Proximity: This destroy operator aims at increasing the chance of improvement by considering pairs that are close to a reference pair e_r in both time and space. We define this proximity by calculating

$$\min_{i \in e_r, j \in e \text{ with } e \in E_d^{\text{pair}} \setminus e_r} \{c_{ij} + (dt_j - at_i)\beta, c_{ji} + (dt_i - at_j)\beta\},$$

with c_{ij} representing the cost of connecting trip i and j in the same duty ($c_{ij} = \infty$ if they cannot be feasibly connected) and $dt_j - at_i$ being the time from the end of trip i to the beginning of trip j, which is weighed by a factor β.

Proximity to cancellation: Similar to the previous operator, but instead of a reference pair we compare the proximity to the currently cancelled trips.

ALNS: Repair Operators

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Less frequent choices: The least frequently chosen pairs of the previous iterations are selected.

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$$\sum_{d \in D} \sum_{e \in E_d^{pair}} \sum_{u \in U_e(t)} x_u^e = 1 - q_t \qquad t \in \hat{T}_r^{RTW},\ r \in R,$$

Vehicle-Driver pair Scheduling Compact (with optional time-window arc constraints):

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$$\sum_{(j,i) \in A_e^{VSDS}} x_{ji}^e - \sum_{(i,j) \in A_e^{VSDS}} x_{ij}^e = \begin{cases} -1, & \text{if } i = s^-, \\ 1, & \text{if } i = s^+, \\ 0, & \text{else}, \end{cases} \qquad i \in V_e^{VSDS},\ e \in E_d^{pair},\ d \in D,$$

$$\sum_{d \in D} \sum_{e \in E_d^{pair}} \sum_{(i,j) \in A_d^{VSDS}(q)} x_{ij}^e \leq w_q \qquad q \in Q,$$

$$W_1^{se} \leq \sum_{(i,j) \in A_e^{VSDS}} w_{ij}^s x_{ij}^e \leq W_2^{se} \qquad s \in S_e,\ e \in E_d^{pair},\ d \in D,$$

$$x_{ij}^e \in \{0,1\} \qquad (i,j) \in A_e^{VSDS},\ e \in E_d^{pair},\ d \in D.$$

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$$x_{ki}^e \leq x_{ij}^e \qquad (k,i) \in A_e^{TW}, (i,j) \in A_e^{VSDS}(t), t \in \hat{T}_e^{RTW}, e \in E_d^{pair}, d \in D,$$

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Vehicle-Driver pair Scheduling Set Partitioning:

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$$\sum_{(j,i)\in A_r^{TT}} z_{ji} - \sum_{(i,j)\in A_r^{TT}} z_{ij} = \begin{cases} -1, & \text{if } i = v_r^-, \\ 1, & \text{if } i = v_r^+, \\ 0, & \text{else,} \end{cases} \qquad i \in V_r^{TT},\ r \in R.$$

$$z_{ij} \in \{0,1\} \qquad (i,j) \in A_r^{TT},\ r \in R,$$

Linking constraints TTVSDS:

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$$\sum_{d\in D}\sum_{e\in E_d^{pair}}\sum_{(i,j)\in A_e^{VSDS}(t)} x_{ij}^e = \sum_{(t,k)\in A_r^{TT}} z_{tk} \qquad t \in T_r,\ r \in R,$$

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$$\sum_{d\in D}\sum_{e\in E_d^{pair}}\sum_{u\in U_e(t)} x_u^e = \sum_{(t,k)\in A_r^{TT}} z_{tk} \qquad t \in V_r^{TT},\ r \in R,$$

Vehicle-Driver pair Scheduling Compact (with optional time-window arc constraints):

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$$\sum_{(j,i)\in A_e^{VSDS}} x_{ji}^e - \sum_{(i,j)\in A_e^{VSDS}} x_{ij}^e = \begin{cases} -1, & \text{if } i = s^-, \\ 1, & \text{if } i = s^+, \\ 0, & \text{else,} \end{cases}$$

$$i \in V_e^{VSDS},\ e \in E_d^{pair},\ d \in D,$$

$$\sum_{d\in D}\sum_{e\in E_d^{pair}}\sum_{(i,j)\in A_d^{VSDS}(q)} x_{ij}^e \le w_q \qquad q \in Q,$$

$$W_1^{se} \le \sum_{(i,j)\in A_e^{VSDS}} w_{ij}^s x_{ij}^e \le W_2^{se} \qquad s \in S_e,\ e \in E_d^{pair},\ d \in D,$$

$$x_{ij}^e \in \{0,1\} \qquad (i,j) \in A_e^{VSDS},\ e \in E_d^{pair},\ d \in D.$$

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$$x_{ki}^e \le x_{ij}^e \qquad (k,i) \in A_e^{TW}, (i,j) \in A_e^{VSDS}(t), t \in \hat{T}_e^{RTW}, e \in E_d^{pair}, d \in D,$$

$$x_{ki}^e \le x_{jl}^e \quad (k,i) \in A_e^{TW}, (i,j) \in A_e^{VSDS}(t), (j,l) \in A_e^{PB}, t \in \hat{T}_e^{RTW}, e \in E_d^{pair}, d \in D,$$

Vehicle-Driver pair Scheduling Set Partitioning:

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$$\sum_{u\in U_d(e)} x_u^d = 1, \qquad e \in E_d^{pair},\ d \in D,$$

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$$x_u^d \in \{0,1\} \qquad u \in U_d,\ d \in D,$$

Outline

1. Mathematical Models
2. Adaptive Large Neighborhood Search
3. Constraint Shortest Path Problem
4. ALNS extensions

Constraint Shortest Path Problem

$$
\begin{aligned}
\min \quad & w^{s'} x \\
s.t. \quad & \textstyle\sum_{(j,i)\in A} x_{ji} - \sum_{(i,j)\in A} x_{ij} = \begin{cases} -1, & \text{if } i = s^-, \\ 1, & \text{if } i = s^+, \\ 0, & \text{else}, \end{cases} & i \in V, \\
& \textstyle\sum_{(i,j)\in A} w^s_{ij} x_{ij} \le W^s & s \in S \backslash s', \\
& x_{ij} \in \{0,1\} & (i,j) \in A,
\end{aligned}
$$

Constraint Shortest Path Problem

$$\min \; w^{s'} x$$

$$s.t. \; \sum_{(j,i)\in A} x_{ji} - \sum_{(i,j)\in A} x_{ij} = \begin{cases} -1, & \text{if } i = s^-, \\ 1, & \text{if } i = s^+, \\ 0, & \text{else}, \end{cases} \qquad i \in V,$$

$$\sum_{(i,j)\in A} w^s_{ij} x_{ij} \le W^s \qquad s \in S \backslash s',$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in A,$$

$$\max_{\pi \ge 0} \min \; w^{s'} x + \pi\Big(\sum_{(i,j)\in A} w^s_{ij} x_{ij} - W^s\Big)$$

$$s.t. \; \sum_{(j,i)\in A} x_{ji} - \sum_{(i,j)\in A} x_{ij} = \begin{cases} -1, & \text{if } i = s^-, \\ 1, & \text{if } i = s^+, \\ 0, & \text{else}, \end{cases} \qquad i \in V,$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in A,$$

Constraint Shortest Path Problem

Motivated by Dumitrescu and Boland (2003)

Preprocessing and all pairs shortest path problems to reduce computational burden.

Constraint Shortest Path Problem

$$\min \; w^{s'} x$$

$$s.t. \; \sum_{(j,i)\in A} x_{ji} - \sum_{(i,j)\in A} x_{ij} = \begin{cases} -1, & \text{if } i = v, \\ 1, & \text{if } i = s^+, \\ 0, & \text{else}, \end{cases} \qquad i \in V,$$

$$\sum_{(i,j)\in A} w^s_{ij} x_{ij} \leq W^s - \nu \qquad s \in S \backslash s',$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in A,$$

$$\max_{\pi \geq 0} \min \; w^{s'} x + \pi\left(\sum_{(i,j)\in A} w^s_{ij} x_{ij} - (W^s - \nu)\right)$$

$$s.t. \; \sum_{(j,i)\in A} x_{ji} - \sum_{(i,j)\in A} x_{ij} = \begin{cases} -1, & \text{if } i = v, \\ 1, & \text{if } i = s^+, \\ 0, & \text{else}, \end{cases} \qquad i \in V,$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in A,$$

Constraint Shortest Path Problem

Algorithm 1 Preprocessing

Step 0:
Initialize:
$U_0 = (|V| - 1) \max_{i,j \in A} c_{ij} + 1$, $U = U_0$

Step 1:
Compute Q^c_{0j} for all $j \in V$
if no path from 0 to 1 was found **then**
  if $U = U_0$ **then**
    STOP: the problem is infeasible
  else
    STOP: the path corresponding to U is an optimal solution
else
  if $w^s(Q^c_{0,1}) \leq W^s$ for all $s \in S$ **then**
    if $c(Q^c_{0,1}) \leq U$ **then** $Q^c_{0,1}$ is an optimal solution
    else STOP: the path corresponding to U is an optimal solution
  else
    if $Q^c_{0,1} \leq U$ **then**
      STOP: the path corresponding to U is an optimal solution
    else $L = Q^c_{0,1}$
Step 2:
for $s \in S$ **do**
  Calculate $Q^{LR(w^s)}_{0j}$ for all $j \in V$
  if $w^s(Q^{LR(w^s)}_{0,1}) > W^s_2$ **then**
    if $U = U_0$ **then** STOP: the problem is infeasible
    else STOP: the path corresponding to U is an optimal solution
  else if $w^{s'}(Q^{LR(w^s)}_{0,1}) \leq W^{s'}$ $\forall s' \in S$ **then** $U = c(Q^{LR(w^s)}_{0,1})$
Step 3:
for $i \in V \setminus \{0, 1\}$ **do**
  if $Q^{LR(w^s)}_{0,i} + Q^{LR(w^s)}_{i,1} > W^s$ for some $s \in S$ **then**
    delete vertex i as well as its incident arcs
  else if $Q^c_{0,i} + Q^c_{i,1} \geq U$ **then**
    delete vertex i as well as its incident arcs
  for $i \in \{0, \dots, |V| - 1\}$ **do**
    if $w^s(Q^{LR(w^s)}_{0,i}) + w^s_{ij} + w^s(Q^{LR(w^s)}_{j,1}) > W^s$ for some $s \in S$ **then**
      delete (i, j)
    else if $c(Q^c_{0,i}) + c_{ij} + c(Q^c_{j,1}) > U$ **then** delete (i, j)
    else if $w^s(Q^c_{0,i}) + w^s_{ij} + w^s(Q^c_{j,1}) \leq W^s$ for all $s \in S$ **then**
      $U = c(Q^c_{0,i}) + c_{ij} + c(Q^c_{j,1})$
Step 4:
if the graph or U was changed in Step 2 or 3 **then** go to step 1
else STOP

Constraint Shortest Path Problem

Algorithm 2 Label Setting Algorithm

Step 0:
Run Algorithm 1 to obtain $Q^{c}_{i,1}$ and $Q^{LR(w^s)}_{i,1}$ for all $s \in S$ and $i \in V \backslash \{1\}$

Initialize:

$L_0 = \{(\mathbf{0}, 0)\},\ L_i = \emptyset$ for all $i \in V \backslash \{0\}$

Step 1:

for $j \in V \backslash \{1\}$ in topological order **do**
  for $(i, j) \in A$ **do**
    for labels $(\mathbf{w}(\mathbf{Q}^{\mathbf{l_i}}), c(Q^{l_i})) \in L_i$ **do**
      if $w^s(Q^{l_i}) + w^s_{i,j} + w^s(Q^{LR(w^s)}_{j,1}) \leq W^s$ for all $s \in S$ and $c(Q^{l_i}) + c_{ij} + c(Q^{c}_{j,1}) < U$ **then**
        Add the corresponding label to L_j while maintaining a lexicographic order
      if $w^s(Q^{l_i}) + w^s_{i,j} + w^s(Q^{c}_{j,1}) \leq W^s$ for all $s \in S$ **then**
        $U = c(Q^{l_i}) + c_{ij} + c(Q^{c}_{j,1})$
    Remove dominated Labels from L_j

Outline

ALNS extensions

Retiming (Veelenturf et al. (2012), van Lieshout et al. (2018))

For trip that is cancelled after an iteration of ALNS, we introduce shifting opportunities. If all cancelled trips already have shifting copies, we allow shifting for trips within a small proximity of cancelled trips.

Trip Merging (Gintner et al. (2005), Sevim et al. (2020))

Trips that are performed in succession on the same vehicle and driver pair in multiple iterations of ALNS are merged to a single trip. This only considered trips of the destroyed part of the solution.

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