A subgraph-sampling heuristic for max-cut

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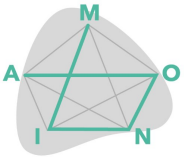
ESR days 2020



Motivation

Statistical Physics

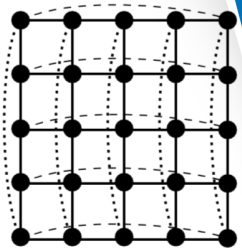
Ground state of spin glasses under the Ising model

Under this model the Hamiltonian of the system is defined by

$$H = -\sum_{\langle i,j \rangle} J_{ij}\sigma_i\sigma_j$$

- σ_i is the i-th spin.
- J_{ij} is the interaction energy between the i-th and the j-th particles.

The goal is to find the lowest energy state.



Max-cut

Max-cut

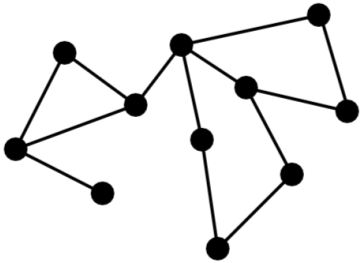
Given $G = (V, E, w)$ the *max-cut problem* calls for a partition $(W : V \setminus W)$ of the node-set defining a maximal-weight edge-cut.

$$\max_{x \in \{-1,1\}^V} \frac{1}{2} \sum_{ij \in E} w_{ij}(1 - x_i x_j)$$

- $x_i = 1 \cdot \chi_{i \in W} - 1 \cdot \chi_{i \in V \setminus W}$

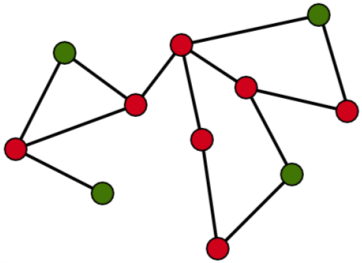
Algorithms: Subgraph sampling scheme

Subgraph sampling scheme



1. Select randomly a point $\hat{x} \in \{-1, 1\}^V$.

Subgraph sampling scheme



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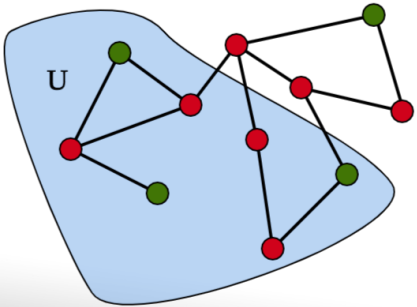
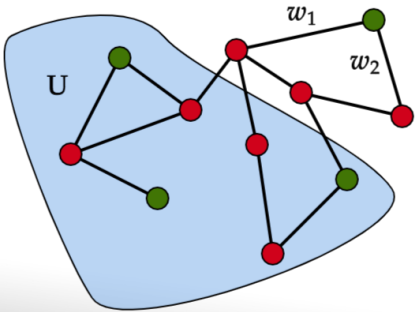


2. Select a “suitable” set $U \subset V$ contracting the nodes $V \setminus U$.

Subgraph sampling scheme



2. Select a “suitable” set $U \subset V$ contracting the nodes $V \setminus U$.
   - Apply *switching* to the nodes in $V \setminus U$ if needed.

$$\hat{x}_i \leftarrow -\hat{x}_i$$

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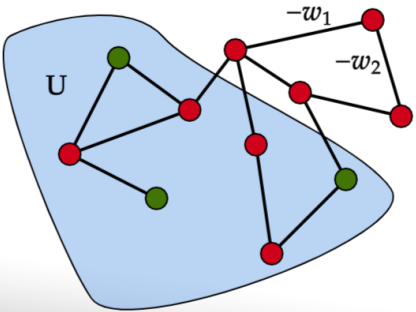

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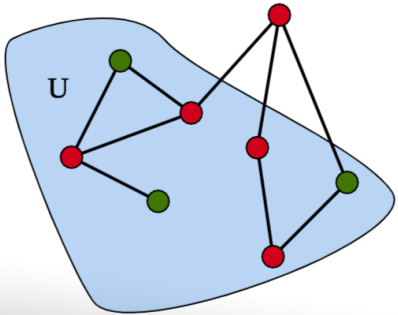

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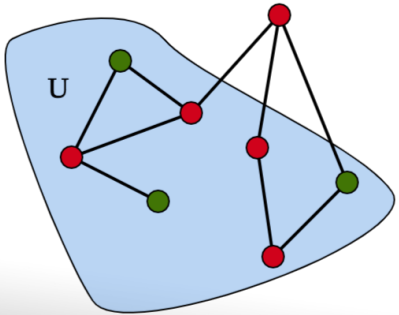
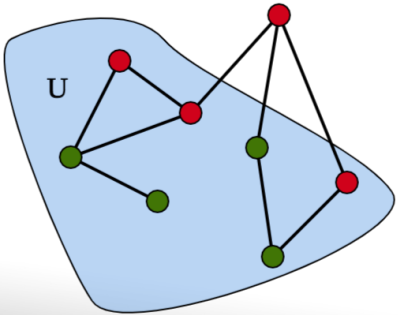


3. Solve max-cut over the "contracted" graph $G_{V \setminus U}$.

Subgraph sampling scheme



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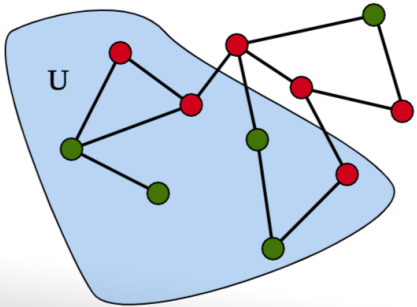

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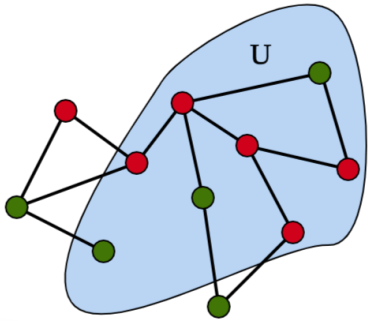
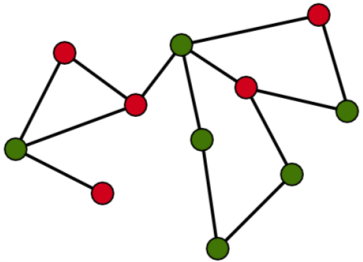


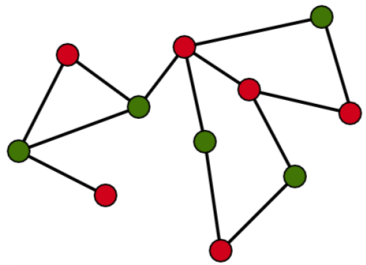
4. Until a maximum number of non-improving iterations is not reached, select a new node-set $U \subset V$ and go to step 2.

Subgraph sampling scheme



5. Perturb the current vector $\hat{x}$ and repeat from step 2. until no more improvements take place.

Subgraph sampling scheme



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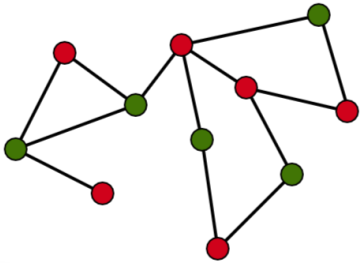
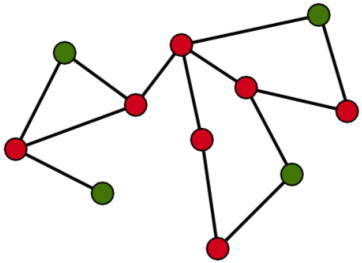


6. If additional computing time is allowed, the node assignment is randomly generated afresh and the process is restarted from step 2.

Subgraph sampling scheme



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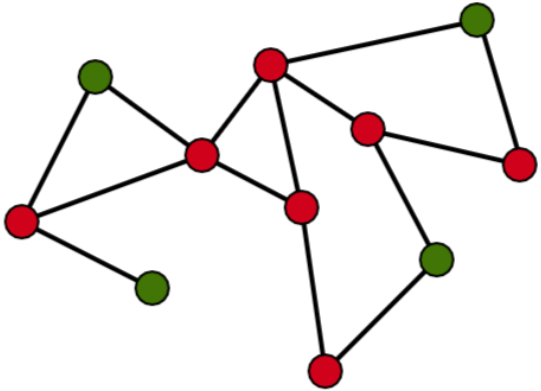
Algorithm: How to select U?

Algorithm: Negative subgraph

Negative subgraph



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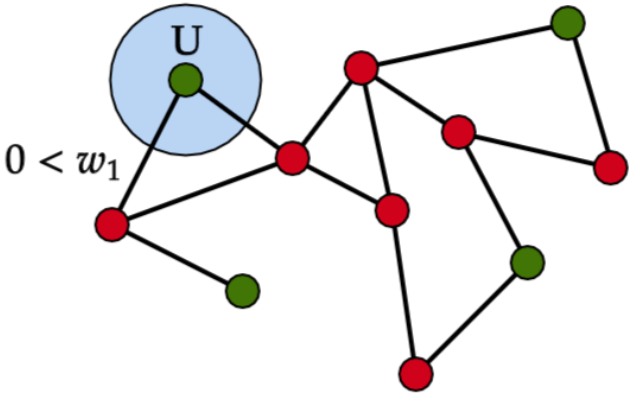

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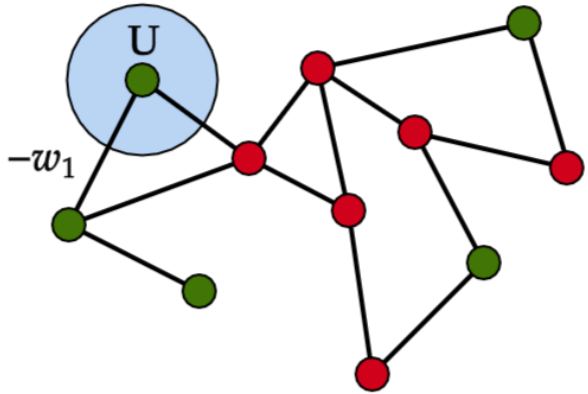

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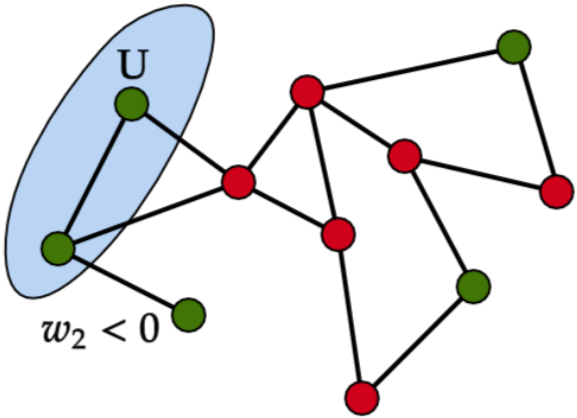

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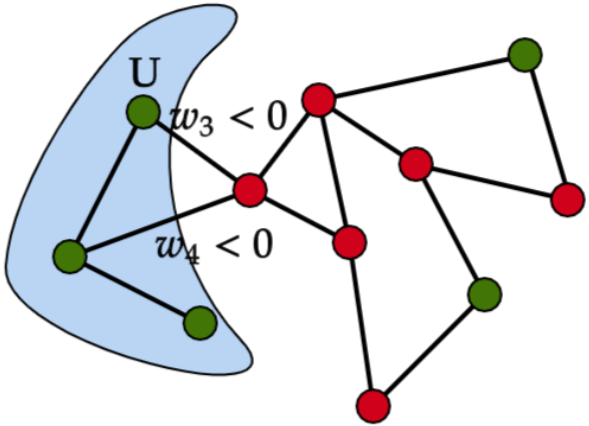

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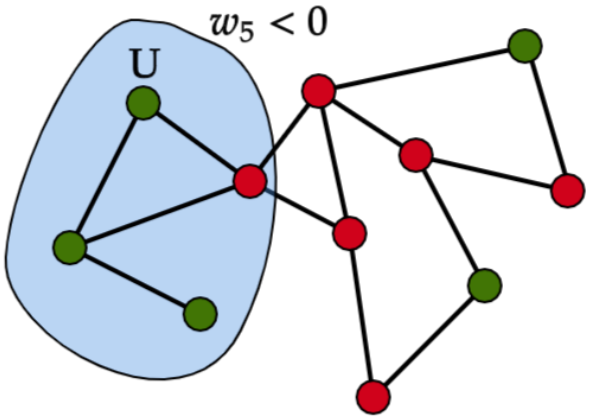

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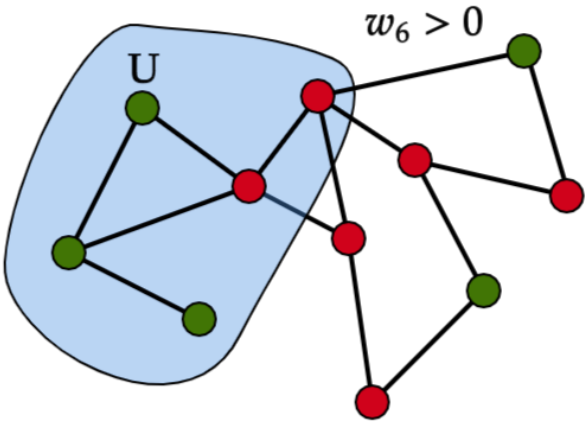

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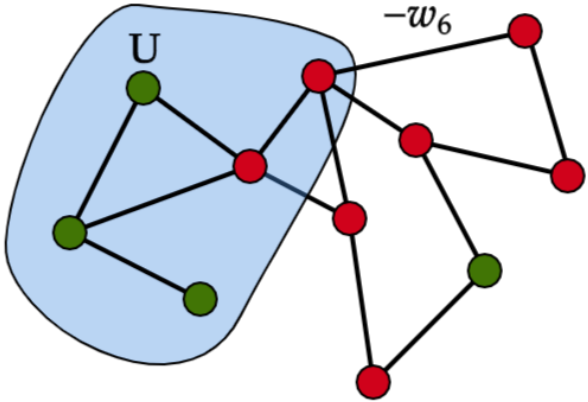

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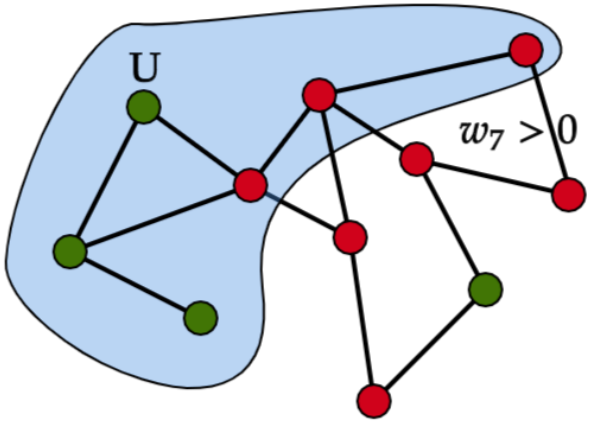

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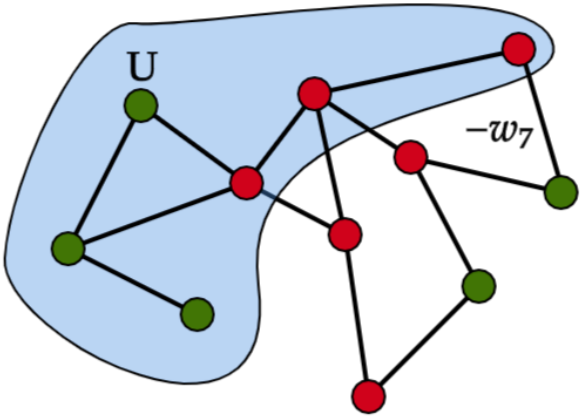

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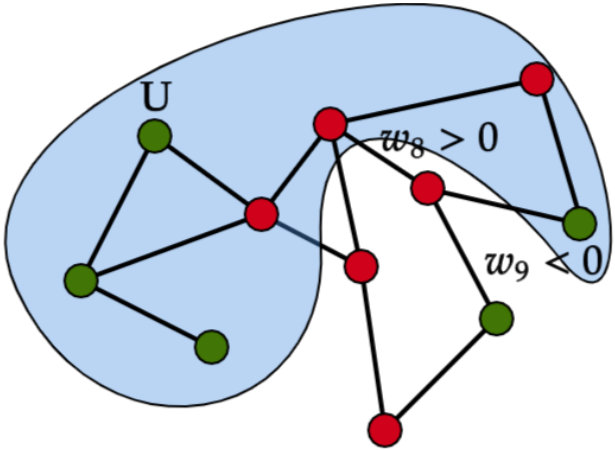

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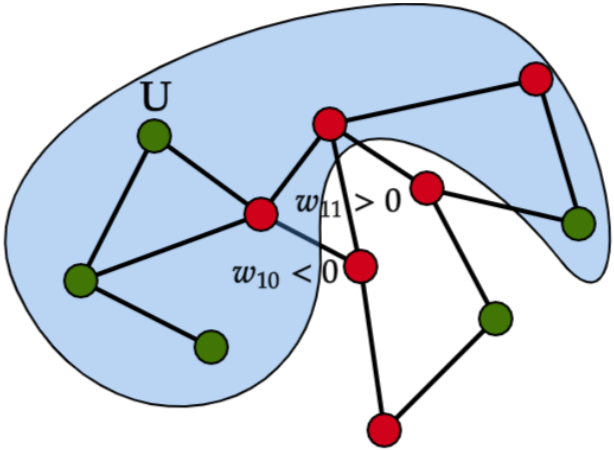

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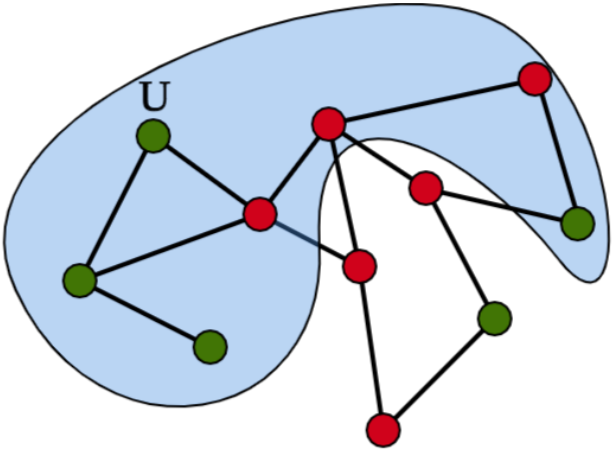
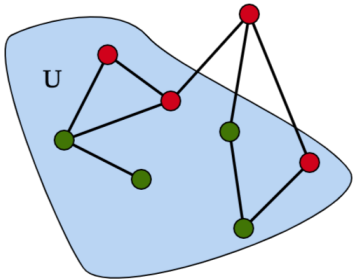


Negative subgraph

Theorem (S.T. McCormick, M.R. Rao and G. Rinaldi, 2003)
For graphs with all the non-negative-weighted edges adjacent to a single node, max-cut is solvable in polynomial time.



Numerical results

Results - Chimera graphs

instance	number	nodes	edges	optimal	time (s)
lCC_nC	150	2049	8064	**150**	<0.01
nC	885	2049	8064	**885**	<0.01
mis	20	2049	8064	**20**	<0.1
random	80	2049	8064	**80**	<2.2
table	100	513	1984	**100**	<0.5
afi	20	2049	8064	0	>4
c8mgw	20	2049	8064	**20**	<2.1
c8selby	20	2049	8064	**20**	<3.9
c16mgw	20	2049	8064	**20**	<7.9
c16selby	20	2049	8064	0	>4
g_a_i_s	30	2049	8064	17	<14
maxcut	80	2049	8064	**80**	<2.9

Results - Biqmac

instance	number	nodes	edges	optimal	time (s)
g05	30	60 - 100	885 - 2475	**30**	<0.7
pm1s	20	80 - 100	316 - 495	**20**	<0.03
pm1d	20	80 - 100	3128 - 4901	**20**	<0.7
w	30	100	495 - 4455	**30**	<1.3
pw	30	100	495 - 4455	**30**	<1.7
ising	30	100 - 300	4950 - 44850	0	>4
toroidal	18	100 - 343	200 - 1029	**18**	<2.7

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