ESR Days 2021: March 4th – March 5th

Day 1

09:15 — 09:30	Opening Remarks
09:30 — 11:20	Lecture: *Multiobjective mixed integer nonlinear programming: decision and criterion space search algorithms* by Marianna De Santis
11:30 — 13:00	Lecture: *It's a Bird... It's a Plane... It's SMS++* by Antonio Frangioni
12:45 — 14:00	Lunch Break
14:30 — 15:30	Tutorial: *Coluna.jl, a branch-and-price-and-cut framework in Julia* by Guillaumes Marques
15:30 — 16:30	Lecture: *On Mixed-Integer Second-Order Cone Optimization: Disjunctive Conic Cuts and Applications* by Tamás Terlaky

Day 2

09:30 — 10:30	Industrial lecture: *Binary Decision Diagrams and Discrete Relaxations of Integer Programming Problems* by Utz-Uwe Haus from HPE
10:30 — 12:30	Lecture pt 1: *Polynomial optimization with the Lasserre hierarchy* Lecture pt 2: *Solving numerically polynomial optimization problems* by Didier Henrion
12:45 — 14:00	Lunch Break
14:00 — 16:00	Tutorial: *Introduction to SCIP* by SCIP team

Abstracts of lectures

Multiobjective mixed integer nonlinear programming: decision and criterion space search algorithms

Most real-world optimization problems in the areas of applied sciences, engineering and economics involve multiple, often conflicting and nonlinear, goals. In the mathematical model of these problems, under the necessity of reflecting discrete quantities, logical relationships or decisions, integer and 0-1-variables need to be considered. We are in the context of MultiObjective Mixed Integer Nonlinear Problems (MO-MINLPs).

The design of efficient solution methods for MO-MINLPs is a big challenge for people working in optimization, as these problems combine all the difficulties of both multiobjective problems and mixed integer nonlinear programming problems. In this lecture, we will give an overview on existing approaches for solving MO-MINLPs. We will present methods belonging to the two main classes of algorithms for MO-MINLPs: decision space search algorithms (methods that work on the decision variables space) and criterion space search algorithms (methods that work in the space of the objectives).

We will show the correctness of the algorithms presented in terms of detecting both the efficient and the nondominated set of MO-MINLPs. Numerical experiments on biobjective and triobjective instances will be presented.

It's a Bird... It's a Plane... It's SMS++

Recently the Structured Modelling System++ has first been released to general public availability, albeit still in an early beta stage. SMS++ is a C++ library intended to facilitate the development of solution methods for very large optimization problems with multiple nested heterogeneous structure, chiefly (but not exclusively) ones based on decomposition. In the attempt of achieving this goal SMS++ has accrued a number of features that look quite unique in the landscape of modelling systems, so much so as to raise the legitimate suspicion that the reason why these features have never been developed before is because no sane person would have ever thought them a good idea. Yet the system is there and it does seem to offer some new viewpoints on some common activities in the mathematical optimization community that may at least be worth a thought. SMS++ developed in an highly modular fashion itself and already counts a(n hopefully growing) number of separate sub-projects besides the "core" library and the support tools. One of these already allows to solve Lagrangian Duals of complex integer programs with remarkable ease, and it will hopefully soon be joined by a similar component doing Benders' decomposition. Hence, there may actually be a few use cases in which SMS++ could be worth considering, notwithstanding the arguably insane delusions of an all-conquering modelling system that some of the developers harbour and that would require capturing an unfeasibly large amount of mindshare to achieve.

On Mixed-Integer Second-Order Cone Optimization (MISOCO): Disjunctive Conic Cuts (DCCs) and Applications

The use of integer variables naturally occurs in Second Order Conic Optimization problems, just as in linear and nonlinear optimization, thus the need for dedicated MISOCO algorithms and software is evident. This talk gives some insight into the design of DCCs for mixed-integer Conic Linear Optimization problems, and into the complexity of identifying disjunctive conic cuts. The novel

DCCs may be used to develop Branch-and-Cut algorithms for MISOCO problems. Several papers prove the power of DCCs in solving in financial optimization layout, and other MISOCO problems. Recent developments include novel, efficient warm start strategies, the identification of pathological disjunctions, and the identification of the optimal partition

Binary Decision Diagrams and Discrete Relaxations of Integer Programming Problems

Binary Decision Diagrams have been used widely to analyze the state space of electronic circuits. Since about decade ago they have been applied to constraint programming, and shown to permit elegant formulation of the feasible set of some common combinatorial problems. We introduce the formalism, showcase applications to combinatorial optimization problems, and discuss how to use it to obtain discrete relaxations in Integer Programming.

Polynomial optimization with the Lasserre hierarchy — Solving numerically polynomial optimization problems

Pt 1: Polynomial optimization with the Lasserre hierarchy

This is an elementary introduction to the Lasserre hierarchy applied to polynomial optimization. We show how the non-convex non-linear problem of minimizing a multivariate polynomial subject to polynomial inequality constraints can be reformulated as a family of convex semidefinite optimization problems of increasing size. Instrumental to this reformulation is the duality between the cone of positive polynomials and the cone of moments of positive measures, and the approximations of these cones with sums of squares certificates of positivity.

Pt 2: Solving numerically polynomial optimization problems

With the help of numerical examples using the Matlab parser GloptiPoly interfaced with the conic solver MOSEK, we illustrate how polynomial optimization problems can be solved efficiently with the Lasserre hierarchy. We describe certificates of global optimality based on flat extensions of moment matrices, as well as approximate recovery of the variety of global minimizers with the Christoffel-Darboux polynomial.